Doing Things With Derivatives

Math 130 - Essentials of Calculus

8 March 2021

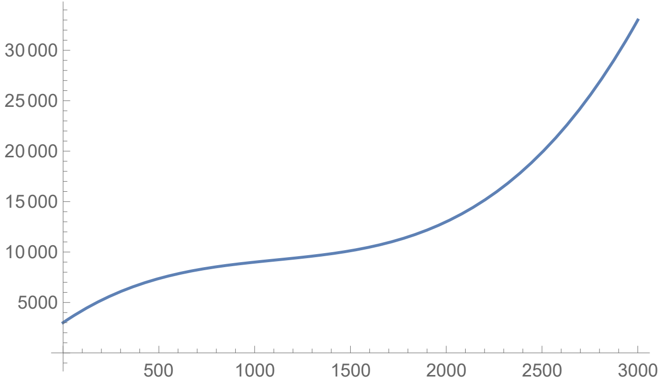
Example

Suppose a company has estimated that the cost, in dollars, of producing q items per week is $C(q) = 3000 + 13q - 0.01q^2 + 0.000003q^3$.

1. What are the fixed costs? \$3000
2. Find a function for the average cost of each unit being produced. What is the average cost when 1500 items are produced? \$6.75
3. Find the marginal cost function. What is the marginal cost when 1500 units are produced? \$3.25
4. What is the actual cost of the 1501*st* item? \$3.25

Minimizing Average Cost

Below is the graph of the cost function from the last example



We can see here that for lower production levels, the cost increases, but at a rate which is decreasing, so that marginal costs are decreasing. However, eventually an inflection point is reached where the marginal cost starts increasing again.

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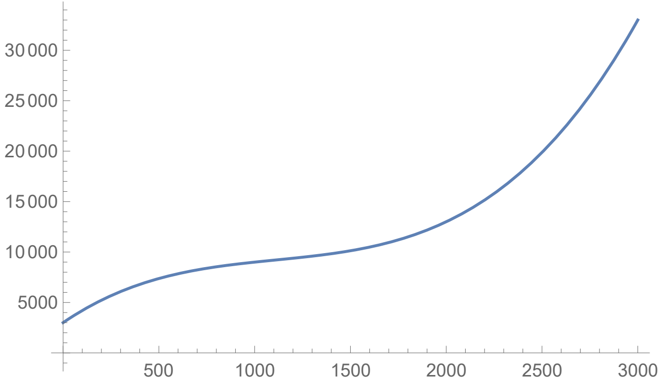

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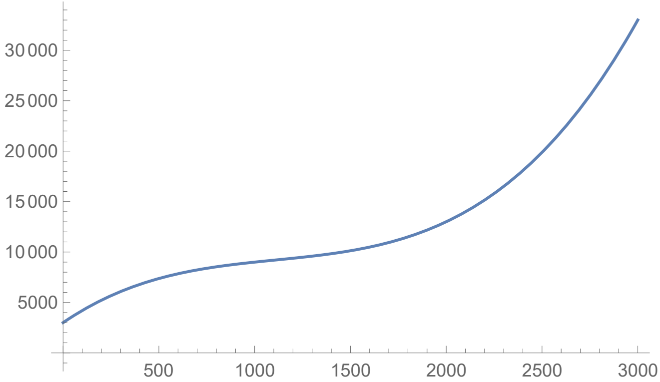
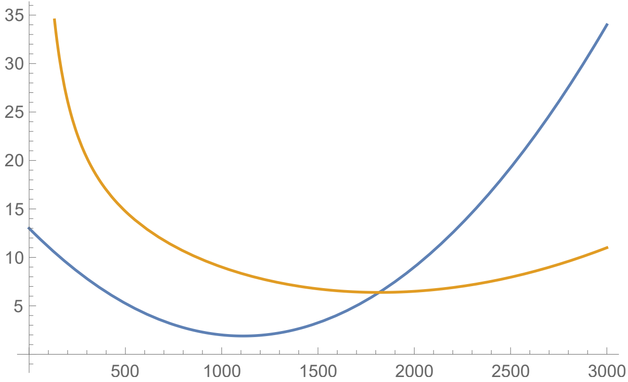


We can see here that for lower production levels, the cost increases, but at a rate which is decreasing, so that marginal costs are decreasing. However, eventually an inflection point is reached where the marginal cost starts increasing again. If marginal cost is increasing, does it make sense to increase production? As long as the marginal cost is less than the average cost, then yes.

Minimizing Average Cost

Below is the graph of the marginal (in blue) and average (in orange) cost function from the last example



It makes sense because if the marginal cost is less than the average cost, producing an additional unit will lower the average cost. Conversely, if the marginal cost becomes higher than the average cost, producing an additional item will increase the average cost.

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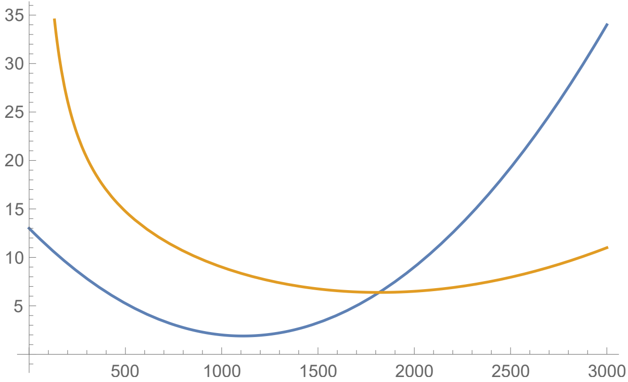

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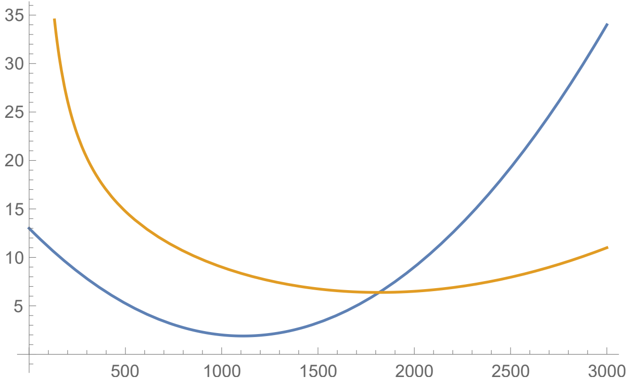

Typically, *the minimum average cost per item occurs when the marginal cost is the same as average cost*, as the previous graph will show.

Example

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A small furniture manufacturer estimates that the cost, in dollars of producing q units of a particular chair each month is given by

$$C(q) = 10000 + 5q + 0.01q^2.$$

How many chairs should be produced in order to minimize the average cost of each chair?

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Example

A baker estimates that it costs

$$C(q) = 0.01q^2 + 2q + 250$$

dollars each day to bake q loaves of bread. How many loaves should be baked daily in order to minimize the average cost?

Revenue and Profit

Revenue, denoted by $R(q)$, is the total amount of money collected by a company after producing and selling q items, and we call R the *revenue function*.

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Definition (Average Revenue and Marginal Revenue)

If $R(q)$ is the total revenue after producing q units of a good or service, then the average revenue per unit is

$$\frac{R(q)}{q}$$

and the marginal revenue is

$$R'(q) = \frac{dR}{dq}.$$

Revenue and Profit

Marginal revenue can be though of as approximately the additional income gained by producing and selling one additional unit (assuming the number of units produced is relatively large). If a producer always charges the same price for each unit of a product, the marginal revenue is always the same (and, in fact, is equal to the price of the object). However, with changing amounts of production, it is typical to change the price. For example, if a significantly larger number of items are produced, oversaturation can occur, driving prices down.

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Regardless, an important question to ask is "how does a company maximize its profits?"

Definition (Profit Function)

The profit function is given by

$$P(q) = R(q) - C(q).$$

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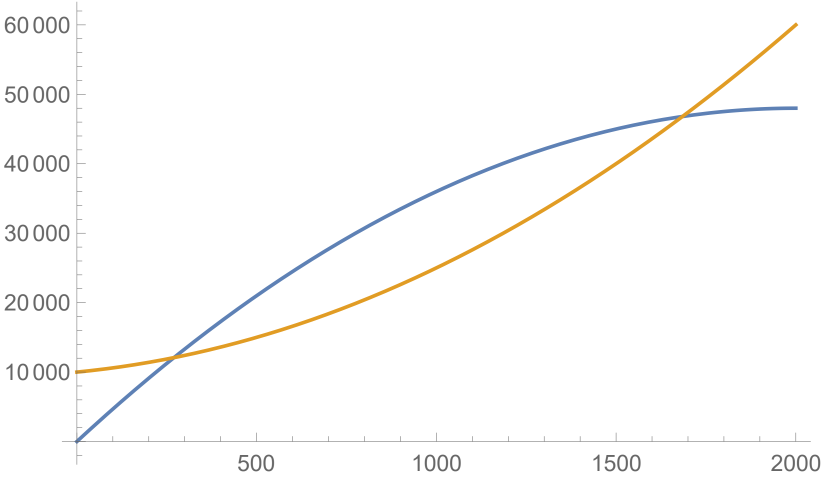
Example

In the situation of the furniture manufacturer example, suppose the company estimates that the revenue, in dollars, realized by producing q units of the chair, up to a maximum of 2000 *chairs is given by* $R(q) = 48q - 0.012q^2$*. (The cost function was given by* $C(q) = 10000 + 5q + 0.01q^2$*.)*

1. *What is the marginal revenue when* 1500 *chairs are produced?*
2. *Determine the number of chairs that the company should produce in order to maximize profit.*

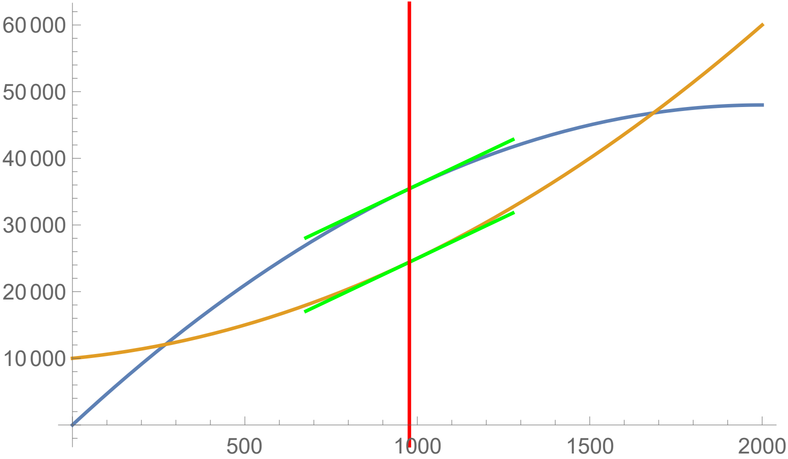
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue).



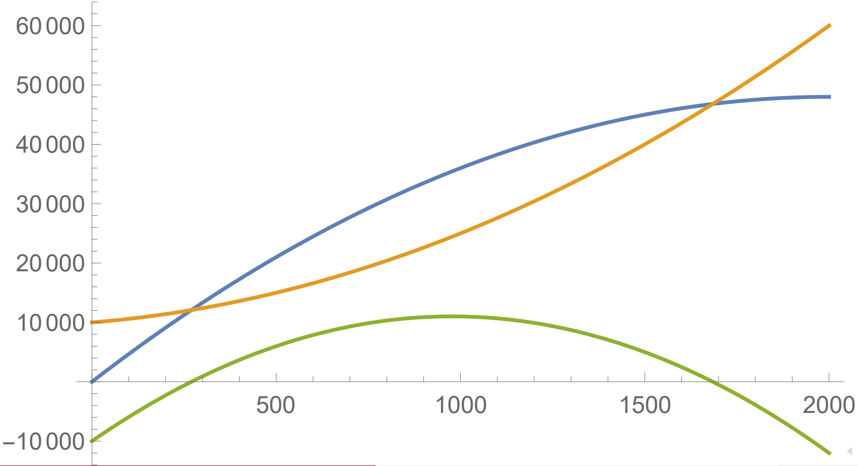
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with their tangent lines (in green) at the number we found before (in red).



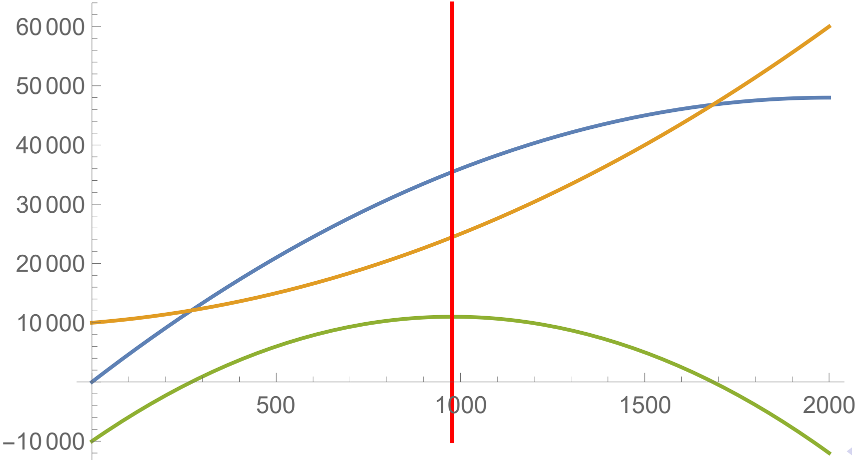
Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with the profit function (in green).



Revenue and Profit

Here is the graph of the cost function (in orange) and revenue function (in blue), together with the profit function (in green) and the red line at $q = 977$.



Now You Try It!

Example

A manufacturer of power supplies estimates that it will incur a total cost of $C(q) = 2500 + 4q + 0.005q^2$ when producing q power supplies, and it will collect $R(q) = 16q - 0.002q^2$ dollars in revenue.

1. *Write a function for the profit P the manufacturer can expect after producing q power supplies.*
2. *Find the marginal cost and marginal revenue functions.*
3. *How many power supplies should the manufacturer produce in order to maximize profit.*

Demand Curves

There is normally a relationship between the price of a product or service and the number of units that can be sold. Let $p = D(q)$ be the price per unit that a company can charge if it sells q units. This function D is called the **demand function** (also called a *price function*) and its graph is called the **demand curve**.

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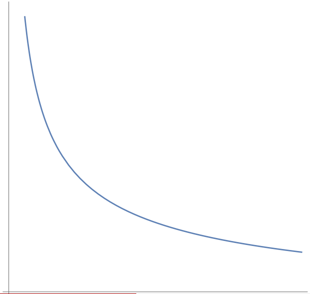

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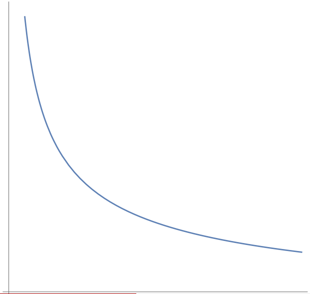

Because revenue is the number of units sold times the price per unit, the revenue can be found as

$$R(q) = q \cdot D(q).$$

Maximizing Profit

Example

A company has cost and demand functions

$$C(q) = 84 + 1.26q - 0.01q^2 + 0.00007q^3 \quad \text{and} \quad D(q) = 3.5 - 0.01q.$$

1. *If the price of each unit is $1.20, how many units will be sold?*

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1. *If the price of each unit is \$1.20, how many units will be sold?*
2. *Determine the production level that will maximize profit for the company.*

Now You Try It!

Example

A company has cost and demand functions

$$C(q) = 680 + 4q + 0.01q^2 \quad \text{and} \quad p = 12 - \frac{q}{500}.$$

Find the production level that will maximize profit.